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WEIGHTED MONTE CARLO CALCULATIONS OF HADRONIC CASCADES IN THICK TARGETS

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I. Introduction

The development of the internuclear cascade which occurs when high energy hadrons interact in thick targets has been examined from many viewpoints; radiation physics: shielding, activation of materials, particle physics: cosmic rays, ionization calorimeters, space research: shielding of spacecraft, activation of meteorites and lunar surface, to name a few. Efforts to interpret these phenomena include both analytical and Monte Carlo (MC) calculations of the cascade development. A recent article[1] written mainly from the shielding standpoint reviews the present status of such calculations and lists the more recent key references. References to earlier work are given in numerous places[2], while the pioneering efforts are described at length by Rossi[3].

Analytical calculations have been mainly confined to one dimension in addition to other simplifying assumptions and quickly become unwieldy when applied to more realistic situations. While MC calculations are readily adapted to a three dimensional treatment of the cascade and the use of more

detailed physical models there are nonetheless several drawbacks. In particular for high energy problems present MC calculations demand large amounts of computer time, storage and coding; even more so for very large targets or for targets with complex geometry or with heterogeneous materials. Also, at high energy the choice of a particle production model for hadron-nucleus interactions has been limited to those readily allowing random sampling of angles and momenta of the outgoing particles.

In this note a MC calculation differing from the conventional ones is described. Advantage is taken of weighting techniques to shorten and simplify the calculations, although at the expense of making it less useful to study fluctuation problems. Similar methods have been used in various other problems (see e.g., Ref. 4), but their joint implementation into hadronic shower simulations requires some elaboration. Below the method is outlined and contrasted with conventional calculations. A brief description of CASIM - a program based on these techniques - is given; by way of an example a comparison of results with previous work is included. A similar approach to intranuclear cascades has been reported earlier[5].

II. Weighted Monte Carlo Calculations

As described in detail in Ref. 1, conventional MC calculations of hadronic cascades reproduce the experimental situation

rather faithfully in a large computer. As illustrated in Fig. 1a, the full genealogy of the cascade is explored and all particles considered carry an equal weight of unity. In the present calculation (CASIM) each generation is represented by a single appropriately weighted particle (Fig. 1b). The weight (see below) is chosen so that the derived quantities such as star density, particle fluxes and energy deposition are reproduced on the average, i.e. over many incident particles. This results in considerable savings in computer time, storage and coding efforts. In CASIM, the relevant parameters of an outgoing particle in a hadron-nucleus collision viz., its kind (i), momentum (p) and angles (Ω) are chosen from a selection (or importance) function $S(i,p,\Omega)$ and weighted according to an assumed production model represented by an inclusive distribution $\frac{dN}{dpd\Omega}$ i.e.,

$$w(i,p,\Omega) = S^{-1}(i,p,\Omega)\ \frac{dN}{dpd\Omega}(i,p,\Omega). \qquad [1]$$

Since only one particle represents all outgoing secondaries, we require

$$\sum_i \int S(i,p,\Omega)\,dpd\Omega = 1. \qquad [2]$$

This procedure is an adaptation of the standard technique of change of variable to evaluate integrals by MC methods. Repeated use of [1] yields an estimate of the average multiplicity of a hadron-nucleus collision. Likewise, if applied to

a chain (as in Fig. 1b), it will yield the average multiplicity of the cascade. While the choice of $S(i,p,\Omega)$ is in principle arbitrary, it will dictate the type of cascades which are predominantly sampled. This can be exploited by choosing a selection function to fit the problem at hand, e.g. to reproduce a conventional MC calculation $S(i,p,\Omega)$ should closely resemble $\frac{dN}{dpd\Omega}$ except perhaps with some concessions to simplify the selection procedure. For the case where they are exactly proportional (assuming for simplicity one kind of particle only)

$$S(p,\Omega) = \frac{1}{\bar{m}} \frac{dN}{dpd\Omega}(p,\Omega) \qquad [3]$$

where the constant $\frac{1}{\bar{m}}$ ($\bar{m} \equiv$ average multiplicity) ensures [2] to be satisfied. In this case, as in a conventional calculation, all particles carry an equal weight of $\bar{m}$. The weight $\bar{m}$ as opposed to unity reflects the fact that a single particle rather than a number of particles (on the average $\bar{m}$) emerge from a collision in CASIM. If, on the other hand, one wishes to explore a more particular problem, e.g. the albedo of a thick target, it would be advantageous for $S(i,p,\Omega)$ to be larger at larger angles. An example of a particularly simple selection function (though not a very useful one for high energy problems) is

$$S(p,\Omega) = \frac{1}{4\pi p_i} \qquad [4]$$

i.e. an isotropic distribution, uniform in momentum with a cut-off at the incident momentum (p_i). In this case

$$w(p,\Omega) = 4\pi p_i \left(\frac{dN}{dpd\Omega}\right) \qquad [5]$$

and the factor $4\pi p_i$ in [5] can be identified as an integration interval $\Delta\cos\theta.\Delta\phi.\Delta p$ ($\theta,\phi \equiv$ polar, azimuthal angle) showing w to be a direct MC estimate of $\bar{m}$ ($= \int \frac{dN}{dpd\Omega}\, dpd\Omega$).

The need for caution in the choice of a selection function is evident. Essentially it should reflect the a priori knowledge one wishes to bring to bear on the problem. For example in the albedo problem it may be questioned whether the large angles of reflected particles result predominantly from one single large angle production process or from a succession of smaller ones. To obtain valid results with moderate computation the answer may be needed and should be incorporated in a proper selection function.

A second form of weighting introduced deals with calculating the distance to the next interaction (r). In conventional calculations this distance is distributed according to $\lambda^{-1}\exp(-r/\lambda)$ where λ is the interaction length of the particle in the material. This is not always efficient, e.g. if one wishes to study cascades over large distances. In CASIM, r is again chosen from a selection function F(r) and a weight

$$W = F^{-1}(r)\lambda^{-1}\exp(-r/\lambda) \qquad [6]$$

is assigned. For heterogeneous media $r' = r/\lambda$ is determined and the particle is transported in successive small steps of length $\Delta r(<<\lambda)$. After each step a geometry subroutine determines the current medium (j) and $\Delta r/\lambda_j$ is subtracted from r' (until $r'<0$). The first step is determined randomly on the interval $(0,\Delta r)$ to avoid systematic errors.

A third basic modification of the conventional scheme occurs in storing the pertinent information about the simulated cascades. In earlier calculations this again more or less duplicated the experiment, e.g. a simulated nuclear interaction occurring at a particular location in the target is recorded by incrementing an appropriate counter from which eventually the star density for that location is derived. As sketched in Fig. 1b, the processes of cascade propagation and recording information in CASIM are more uncoupled, i.e. for each particle created in the cascade the number of stars along its trajectory are calculated at fixed intervals ($\Delta\ell$) until either (i) it escapes the target, (ii) the length of the trajectory exceeds its range, (iii) its weight is less than a predetermined cut-off, i.e. when it no longer contributes useful information; an identical weight cut-off is employed to terminate the cascade itself. The distance $\Delta\ell$ is chosen to be of the order of either λ or the spatial resolution of the calculation, whichever is smaller. Again

the first interval along the trajectory is determined randomly (Fig. 2). A similar technique is used for energy deposition.

The above is summarized in a simplified flow chart of CASIM (Fig. 3). Some details were omitted from Fig. 3 for brevity: the treatment of π^0 differs from that of conventional calculations where they are essentially on equal footing with the other hadrons. Because of their short lifetime, π° do not propagate the cascade (but are important by-products carrying off a large share of the energy) it would be inefficient to terminate a cascade every time a π° is selected. Instead, the weight of the created particles (other than π°) is lowered by a factory $(1-\bar{m}_{\pi^\circ}/\bar{m})$ where $\bar{m}_{\pi^\circ}$ and $\bar{m}$ are the π° and total average multiplicity of the parent. The energy deposited by the π° is treated by considering their production from the other hadrons (and the ensuing electromagnetic cascades) in an average sense during the "recording" phase. Some of these features can be directly incorporated in conventional MC calculations. Likewise CASIM employs some of the more familiar techniques, e.g. the method (used by Ranft[1]) of forcing incident particles to interact in the first "slab" and including collisions deeper in the target analytically.

In Fig. 4 the radial dependence of the star density at the shower maximum obtained with Ranft's program FLUTRA[1] is

compared with results of CASIM for the case 29.4 GeV/c protons normally incident on a solid iron beamstop. Both calculations were performed with essentially the same model. The agreement is impressive at least where FLUTRA provides information.

Results of CASIM applied to various realistic shielding configurations including comparisons among different production models are deferred to a future communication.

III. Advantages and Limitations

Some advantages of CASIM have already been mentioned viz., computer economics and using selection functions to fit particular problems. The representation of the cascade by a single member is especially advantageous at higher energies: for conventional calculations computer time spent per incident particle varies approximately linearly with energy, for CASIM it increases roughly logarithmically and allows for a more uniform sampling of the cascades.

It is to be noted, as shown in [1], that the particle production (in the form of an inclusive distribution) is computed directly, thereby allowing CASIM to use quite complicated production models. In particular inclusive distributions from elementary particle collisions have received a great deal of attention recently and there is reasonable expectation that inclusive particle-nucleus spectra may be calculated from them[5].

The direct use of inclusive distributions is to be contrasted with two earlier calculations viz., the programs of Ranft[1,6] and of the ORNL group[7,8]. In the calculations of Ranft outgoing particles are selected from an inclusive distribution via random sampling (i.e. the selection function equals the assumed distribution) in the sense that their spectra will match the distribution on the average. Requirements of energy conservation are introduced either at each interaction (FLUTRA) or in the mean (TRANSK). For results averaged over many incident particles both calculations are equivalent to each other and to CASIM if the production employed there is energy conserving. In the ORNL programs particle production is treated by simulating a fully three dimensional intranuclear cascade. Economical considerations aside there are difficulties at high energies where lack of detailed knowledge of particle production (in the exclusive sense) precludes carrying out conventional intranuclear cascade calculations. Likewise, the use of results for low energy cascades to estimate high energy interactions[8] is questionable for the same reason. A more attractive and more economical approach would be to perform - in the CASIM scheme - an inclusive type intranuclear cascade[5] (when available) to simulate an interaction.

Another form of weighting (particle splitting[4]) is used by Ranft in TRANSK to obtain results over large distances, i.e. the more penetrating particles are made to undergo a number of repeated interactions and the outgoing particles are appropriately weighted. This scheme (in a sense opposite to CASIM) involves more coding and computer time.

As mentioned in the Introduction, the present calculation is less useful in studying fluctuation problems, e.g. analysis of ionization calorimeter data. Conventional calculations can sample the cascade on an event-by-event basis and the required distributions (e.g. energy deposited in a thin layer of scintillator embedded in the target) are readily obtained. Note however, that this will depend on the production model assumed including the sampling procedures which have often been more convenient than realistic.

Techniques similar to those of CASIM could perhaps be used to study fluctuations by estimating the variance (or higher moments) of a distribution. However, since correlations are clearly important in this problem, this may be expected to be much more cumbersome than estimating averages and would negate many of the advantages mentioned. Also, there seems to be, at least in the study of calorimeters, less need for this. Typical dimensions of present calorimeters are well within the limits where conventional calculations

are reasonably efficient and little appears to be gained in resolution by increasing their size.

In summary: CASIM offers a fast and simple way to study the average cascade development in thick targets gaining in usefulness at higher energies. The ability to focus on particular problems and a wider choice in particle production models are further improvements over earlier calculations.

Acknowledgments

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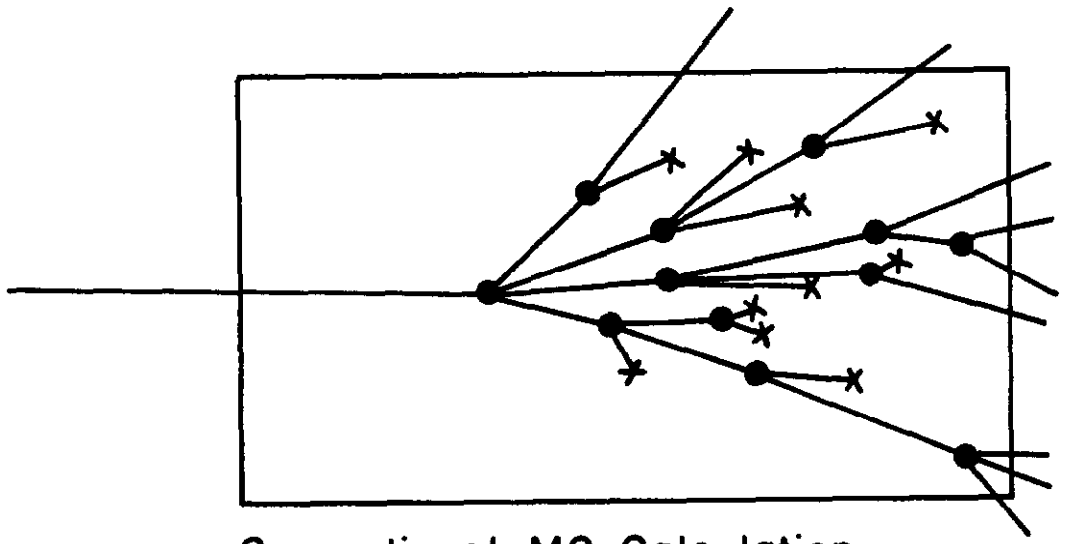
Figure Captions

Fig. 1 Schematic illustration of (a) conventional MC calculation, (b) present calculation. In (b) the solid lines show the simulated propagation of the cascade while the dashed lines are the trajectories along which star production and other information are calculated (for primary particles this is performed analytically).

Fig. 2 Illustrative example of the recording procedure of CASIM. A neutron originates at A and is stepwise transported along its trajectory until it escapes the target. At each point, B_j an amount of star production $w^1 = w[1-\exp(\Delta\ell/\lambda)]\exp(-j\Delta\ell/\lambda)$ is added to an appropriate counter ($w \equiv$ weight at point of origin A, $\lambda \equiv$ interaction length).

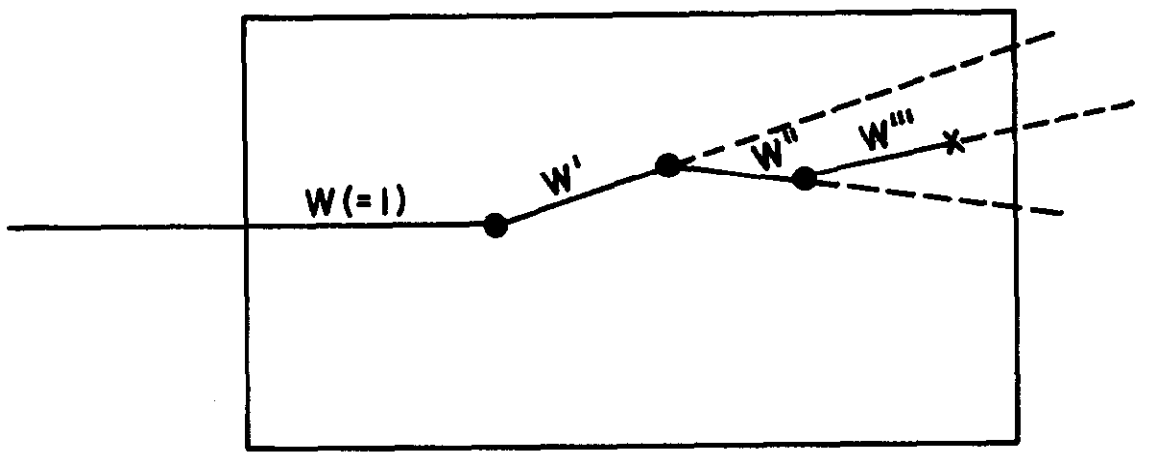
Fig. 3 Simplified flow chart of CASIM.

Fig. 4 Comparison of J. Ranft's MC calculation FLUTRA with present results on the radial dependence of the star density at the shower maximum for the case of 29.4 GeV/c protons normally incident on a solid iron beam dump.



(a)

Conventional MC Calculation



(b)

Present Calculation

Fig.1

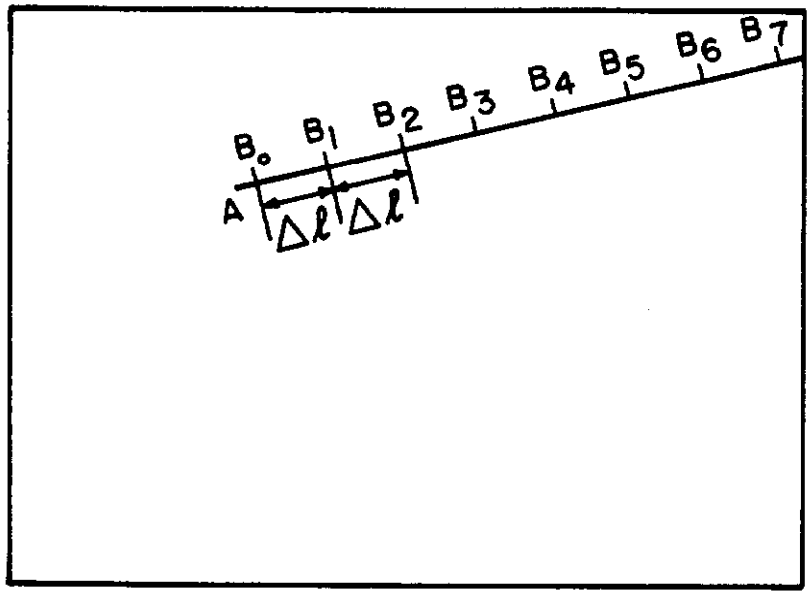


Fig. 2

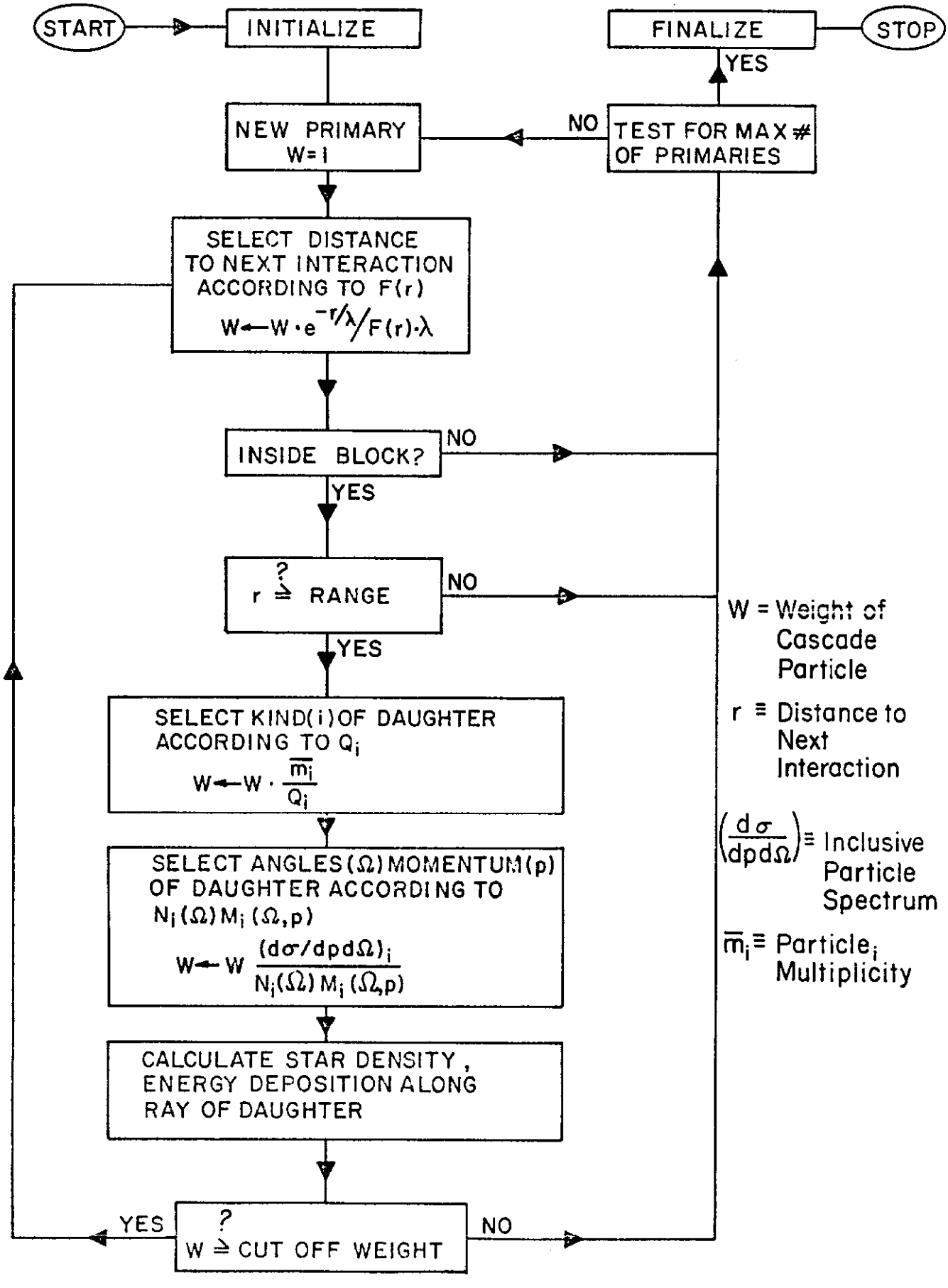

START
INITIALIZE
FINALIZE
STOP
YES
NEW PRIMARY
W=1
NO
TEST FOR MAX # OF PRIMARIES
SELECT DISTANCE TO NEXT INTERACTION ACCORDING TO F(r)
W←W·e^{-r/λ}/F(r)·λ
INSIDE BLOCK?
NO
YES
r ≟ RANGE
NO
YES
SELECT KIND(i) OF DAUGHTER ACCORDING TO Q_i
W←W·m̄_i/Q_i
SELECT ANGLES(Ω) MOMENTUM(p) OF DAUGHTER ACCORDING TO N_i(Ω) M_i(Ω,p)
W←W (dσ/dpdΩ)_i / N_i(Ω) M_i(Ω,p)
CALCULATE STAR DENSITY, ENERGY DEPOSITION ALONG RAY OF DAUGHTER
YES
W ≟ CUT OFF WEIGHT
NO
W = Weight of Cascade Particle
r ≡ Distance to Next Interaction
(dσ/dpdΩ) ≡ Inclusive Particle Spectrum
m̄_i ≡ Particle_i Multiplicity

Fig. 3

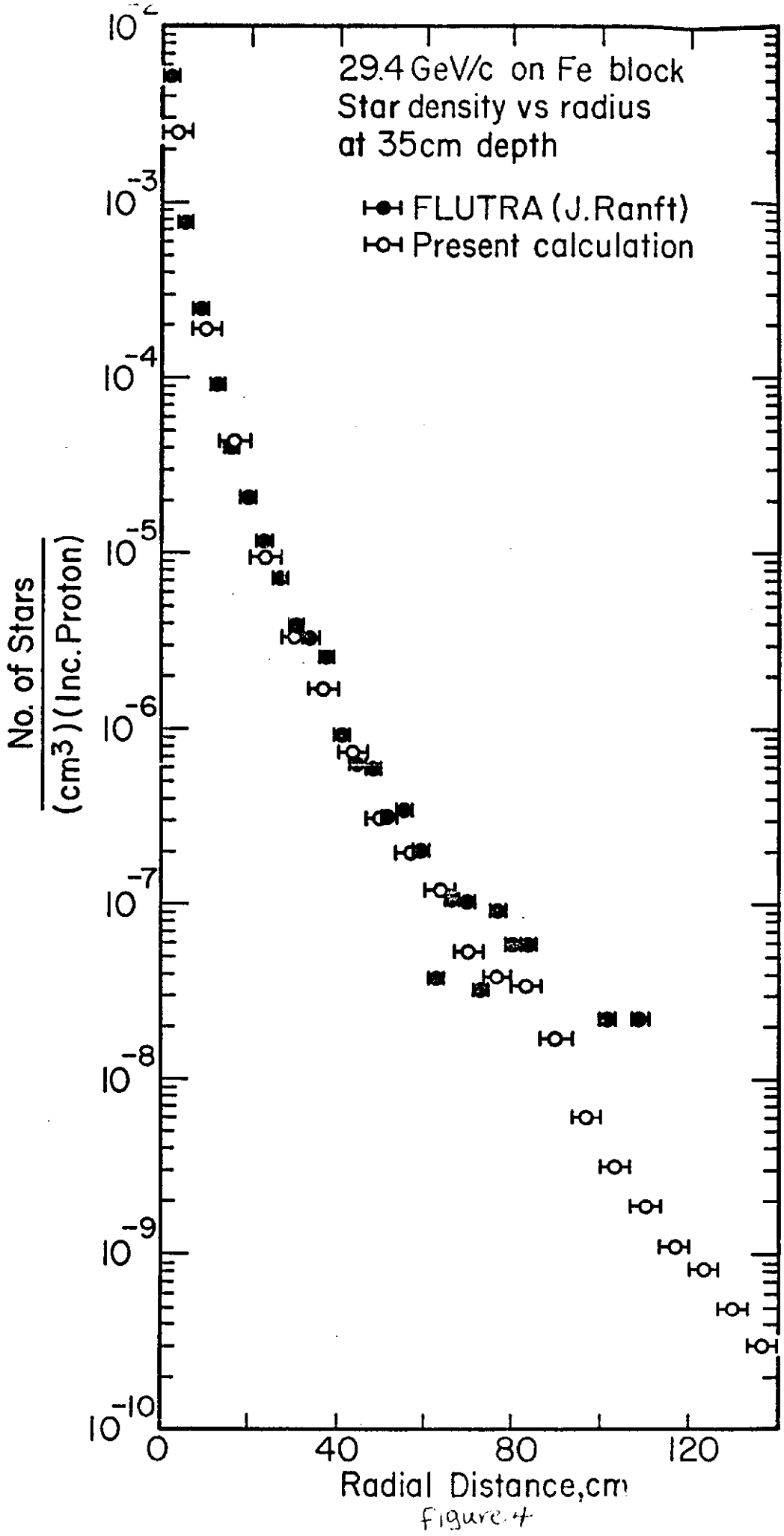

29.4 GeV/c on Fe block
Star density vs radius
at 35cm depth
FLUTRA (J.Ranft)
Present calculation
No. of Stars / (cm³)(Inc. Proton)
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
0
40
80
120
Radial Distance, cm

Figure 4